

## ANALYTICAL THERMAL ANALYSIS ON STRAIGHT TRIANGULAR FINS

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### Abstract

Thermal behaviour of Triangular fin is analytically investigated using first principle. An analytical closed form solution of the problem under-consideration is obtained manually since the governing equations and their boundary conditions are linear. The results of this study show that MATLAB can be used effectively and efficiently to solve challenging heat transfer problems. Heat transfer through various fin geometries is modeled. Geometry configurations such as Triangular profiles are available. The length, base thickness, and end thickness of the fin is specified. Coarse, medium, and fine mesh types are available. Thermal conductivity of the fin material is specified. A constant temperature condition is applied at the base of the fin. Fully insulated and convective boundary conditions can be applied at the tip of the fin. The results report base wall temperature, total area for heat convection, heat dissipation rate, fin efficiency, fin effectiveness and Contours of temperature distribution & heat flux.

**Keywords:** MATLAB, Analytical, Thermal behaviour,

### NOMENCLATURE (General Terms)

$h$	= Convective Heat transfer coefficient, [W/m <sup>2</sup> -k]
$k$	= Thermal conductivity, [W/m-k]
$T_{\infty}$	= Ambient temperature, [k]
$T_0$	= Base temperature, [k]
$dT$	= Temperature difference, [k]
$dx$	= Axial difference, [mm]
$q$	= Heat flux, [W/mm <sup>2</sup> ]
$Q$	= Heat transfer rate, [W]
$x$	= Coordinate
$p$	= Perimeter, [m]
$w$	= Width of the fin, [m]
$L$	= Length of the fin, [m]
$t$	= Thickness of the fin, [m]
$A_s$	= Total convective surface area, [m <sup>2</sup> ]
$A_c$	= Cross- section area, [m <sup>2</sup> ]
$I_0$ & $I_1$	= Bessel functions

### Greek Symbols

$\rho$	= Density, [kg/m <sup>3</sup> ]
$\theta$	= Dimensionless temperature, $(T-T_{\infty}) / (T_0-T_{\infty})$
$\theta_0$	= Excess base temperature, $(T_0 - T_{\infty})$ (°C)
$\theta_t$	= Dimensionless tip temperature, $(T_t - T_{\infty}) / (T_0 - T_{\infty})$

### Subscripts

$0$	= Base
$\infty$	= Infinite
$c$	= Cross-section
$s$	= Surface
$\theta$	= Temperature difference, [k]

### Subscripts

conv	= convection
ch	= channel
sp	= single phase
bot	= bottom

$f$	= fluid
$i$	= inlet
$o$	= outlet

Fins are used to enhance convective heat transfer in a wide range of engineering applications, and offer a practical means for achieving a large total heat transfer surface area without the use of an excessive amount of primary surface area. Fins are commonly applied for heat management in electrical appliances such as computer power supplies or substation transformers. Other applications include IC engine cooling, such as fins in a car radiator. It is important to predict the temperature distribution within the fin in order to choose the configuration that offers maximum effectiveness.

Generally, the fin are used on the surface where the heat transfer coefficient is very low, for example in a car radiator the outer surface of the tube in finned. Because the heat transfer coefficient for air at the outer surface is much smaller than that of water flow inside the tubes. Similarly the electrical transformer and the motor in which the generated heat is dissipated to air by providing fin on its outer surface. The fins are also provided on cylinder and cylinder head of an air cooled I.C. engine and variety of heat exchanger. The selection of fins is made on the basis of thermal performance and cost. The selection of suitable fin geometry requires a compromise among the cost, weight, available shape, pressure drop of the heat transfer fluid and heat transfer characteristics of the extended surface. Kern and Kraus [1] have identified three main fin geometries. These are longitudinal fins, radial or circumferential fins and pin fins or spines. Profile under convective conditions was first proposed by Schmidt [2] based on a physical reasoning. Later on Duff in [3] proved Schmidt criteria using calculus of variation. Both Schmidt [2] and Duff in [3] estimated the fin surface area neglecting the profile curvature. This has formed a major assumption in further exercises of fin optimization and is

known as length of arc idealization (LAI) in literature. LAI was used for optimizing fin shapes under convective, radiating, convective-radiating condition [4], for fins with heat generation [5] and for variable thermal conductivity. Maday [6] in his pioneering analysis proposed the correct formulation for the optimization of longitudinal fin with the elimination of LAI and obtained a profile much different from Duffin [3]. Guceri and Maday [7] further extended this analysis for radial fins. However, fin shapes determined by the above procedure are complex and difficult to manufacture. These fins have structurally weak slender tips, which do not substantially contribute to the overall heat dissipation. This has resulted in a parallel effort to design optimum fins where the fin shape is specified a priori and fin dimensions are determined to give maximum heat dissipation for a given fin volume. Chung and Kan [11] considered the effect of profile curvature on the optimum dimensions of longitudinal fins of triangular, concave and convex parabolic profile. While they have proposed an analytical solution for triangular fin they had to take the resort of numerical techniques for parabolic fins. Razelos and Satya prakash [12] presented an analysis for optimum longitudinal fin of trapezoidal section based on an assumption of negligible heat loss from the fin tip and negligible surface curvature effect and finally suggested a correlation for the optimality criteria. Based on a diameter dependent convective heat transfer coefficient, Chung [13] improved the design of optimum cylindrical pin fins originally proposed by Sonn and Bar-Cohen [14]. Chung and Kan [11] determined the optimum dimensions of spines having different profiles (cylindrical, conical, concave and convex parabolic) from a generalized formulation using a numerical procedure. They reported a profound influence of profile curvature on the optimum dimensions of the spine. Razelos [15] analyzed the heat transfer from convective spines of different profiles assuming negligible surface curvature and no tip loss.

**OBJECTIVES OF THE PRESENT WORK:**

As is evident from the diversity of application areas, the study of heat transfer in Straight Triangular fins is very important for the technology of today and the near future, as developments are following the trend of miniaturization in all fields. Analysis shows that the triangular heat sinks were studied extensively the present work is undertaken to study the following aspects of

- Parameter sensitivity study of straight fins.
- Comparison between materials of triangular fins through analytical value in all aspect.

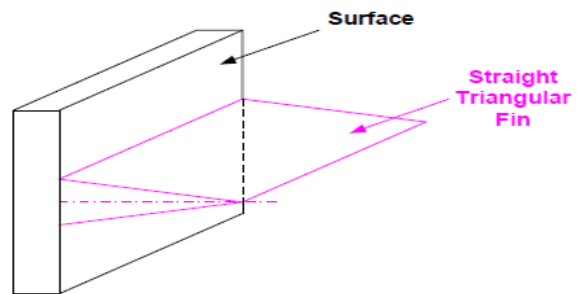
**SPECIFICATION OF PROBLEM**

Consider the first one is straight triangular fins. Whose physical geometry is same, means the length L is .025 meter, width w is .06 meter and thickness t is .004 meter .These fins are made of two different materials which is steel and aluminum, thermal conductivity is 17, 237 w/m-k respectively. The convective heat transfer coefficient is 4

w/m<sup>2</sup>-k. Ambient atmosphere temperature is 30<sup>0</sup>c and fin wall temperature is 400<sup>0</sup>c is constant over the cross-section.

**Assumptions:**

1. Fin has a uniform cross- section area.
2. The fin made of a material having uniform thermal conductivity (K=const.)
3. The heat transfer coefficient between the fin and the fluid is (h=const.)
4. One dimension steady state condition only.
5. No heat generation(q=0)
6. Radiation is negligible.



**Fig 1: Problem Geometry**

**ANALYTICAL SOLUTION OF STRAIGHT TRIANGULAR FIN:**

The straight triangular fin is of practical importance, because it dissipates maximum heat per unit weight. Assuming the sufficient thin fin (L>>t), so the one dimension heat conduction can be considered

$$\frac{d}{dx}(A_c \frac{dT}{dx}) - \frac{h dA_s}{k dx} [T(x) - T_{\infty}] = 0 \tag{1}$$

For a straight triangular fin, the cross-section area varies with fin length. Its value at any x position in terms of length ratio x/L is given by  $A_c = A(x) = w \frac{x^2}{L}$ ,

The perimeter,  $P = 2w$   
 Surface area,  $dA_s = P dx = 2w dx$

Substituting  $A_c$  and  $dA_s$  in energy equation

$$\frac{d}{dx}(w \frac{x^2}{L} \frac{dT}{dx}) - \frac{h 2w dx}{k dx} [T(x) - T_{\infty}] = 0$$

$$\frac{d^2}{dx^2}(T) + \frac{1}{x} \frac{dT}{dx} - \frac{2hL}{k tx} (T - T_{\infty}) = 0$$

Introducing  $\theta = (T - T_{\infty}), \frac{d^2}{dx^2}(\theta) + \frac{1}{x} \frac{d\theta}{dx} - \frac{2hL}{k tx}(\theta) = 0$

Introducing,  $\beta^2 = \frac{2hL}{kt}$  (2)

And multiplying throughout by  $x^2$ , we get

$$x^2 \frac{d^2}{dx^2}(\theta) + x \frac{d\theta}{dx} - \beta^2 x (\theta) = 0 \tag{3}$$

The eqn. (3) is a modified Bessel equation and its solution is

$$(\theta) = C_1 I_0(2\beta\sqrt{x}) + C_2 K_0(2\beta\sqrt{x}) \tag{4}$$

Where  $I_0$  and  $k_0$  are modified zero order Bessel function of the first and second kind, respectively and  $C_1$  and  $C_2$  are constant of integration and are evaluated from boundary condition. At  $x=0$ ,  $\theta = \text{finite}$   $\frac{d\theta}{dx} = 0$ , It gives  $C_2=0$ , At  $x=L$  (at root)  $\theta = \theta_0$ , using  $\theta_0 = C_1 I_0(2\beta\sqrt{L})$  Using in  $C_1$  and  $C_2$  eqn. (4), we get

$$\frac{\theta}{\theta_0} = \frac{I_0(2\beta\sqrt{x})}{I_0(2\beta\sqrt{L})} \tag{5}$$

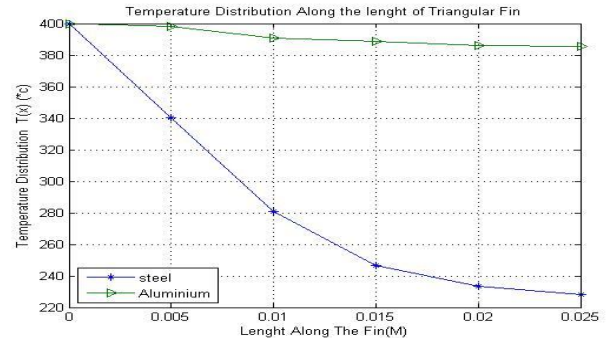
The heat rate can be obtained from the Fourier conduction law and first derivative of eqn. (4) at  $x=L$

### RESULTS AND DISCUSSIONS

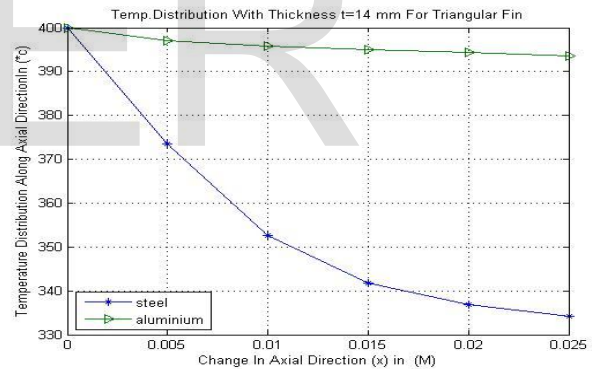
**Fig.2** Resents the temperature distribution along the length of the triangular fin. A higher slope can be observed near the base of the fin due to the maximum temperature difference between fin surface and the surrounding medium at the base. Now replace the value of convective heat transfer coefficient from  $h=40 \text{ w/m}^2\text{-k}$  to  $h=90 \text{ w/m}^2\text{-k}$ , so large amount of temperature drop along the Triangular fin is about  $385.6324 \text{ }^\circ\text{C}$  for Aluminium. In **Fig.3** replace the value of thickness  $t=4 \text{ mm}$  to  $t=14 \text{ mm}$ . then we will find that fin have less amount of temperature drop at every point of the fin. Means fin is less suitable or less utility when we using the high value of thickness and don't allow more thickness of the fin for purpose of heat rejection. **Fig. 4** what effect will occur when change the ambient temperature. Now new value of ambient temperature is  $T_\infty = 10 \text{ }^\circ\text{C}$ . Then we observed that the difference between  $(T_0 - T_\infty)$  is high. Means the temperature distribution gives higher value. The temperature decreasing less as soon as reached to the end position of the fin. For higher temperature drop, should use low value of ambient temperature  $T_\infty$ . In **Fig. 5** Triangular fin is perform well. Aluminium material gives high rate of heat remove from the surface in comparison to Steel material. **Fig.6.** consider the new value of convective heat transfer  $h=90 \text{ w/m}^2\text{-k}$  in place of  $h=40 \text{ w/m}^2\text{-k}$ . Because of new value of  $h$ , the heat transfer rate increases maximum value  $83.3077 \text{ W}$  is achieve for Triangular fin for Aluminium. Steel remove less amount of heat in comparison to Aluminium material. **Fig.7.** It gives the same shape of graph as earlier in figure 6. But it dissipated the large amount of heat because surface area increases very large in both case of fin. In this case, Aluminium material again gives high rate of heat dissipation because of high value of thermal conductivity and Steel material gives less amount of heat dissipation because of less value of thermal conductivity. That means Aluminium material

works well for the fin case. **Fig. 8** what effect will occur when change the fin wall temperature. Now change the value of fin wall temperature  $T_0 = 500^\circ\text{C}$ . Figure shows the heat transfer rate along the length of the Triangular fin.

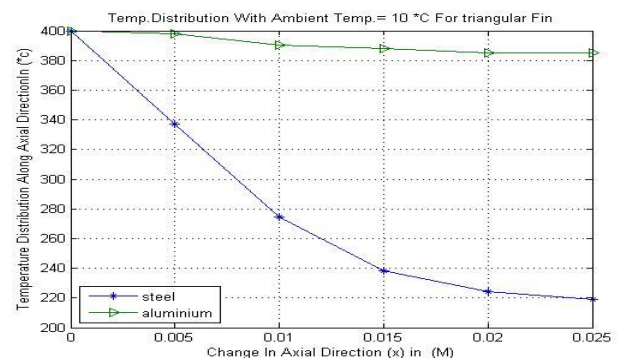
A higher value can be observed near the base of the fin due to the maximum temperature difference between fin surface and the surrounding medium at the base. It is also and heat transfer rate increases as soon as reached to the end position of the fin and Aluminium material again dominated in both case of fin.



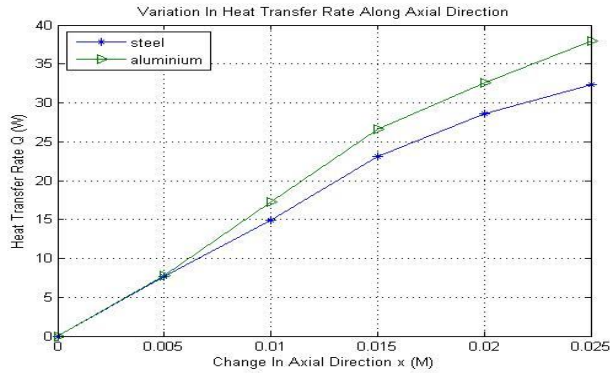
**Figure 2:** Temperature distribution along the different position of fin



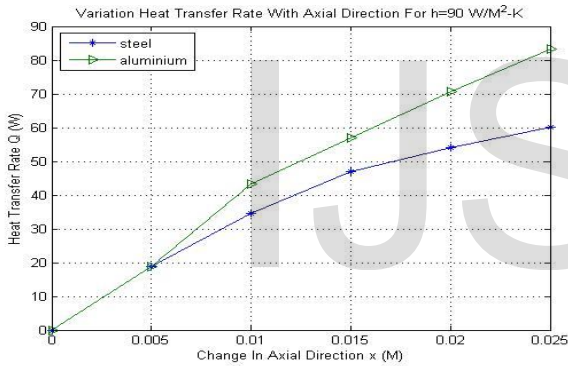
**Figure 3:** Variation in temp. Distribution for along the different position of fins with thickness  $t=14 \text{ mm}$ .



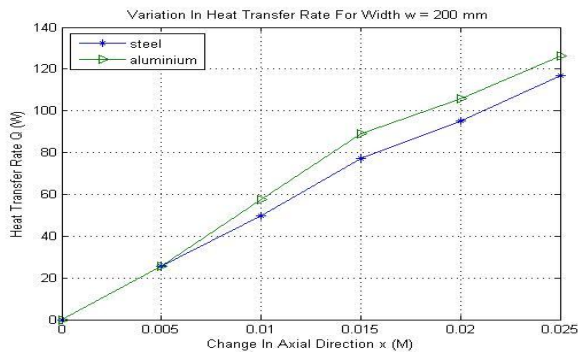
**Figure 4:** Variation in temperature distribution for along different position of fin with ambient temperature  $T_{\infty}=10^0\text{C}$



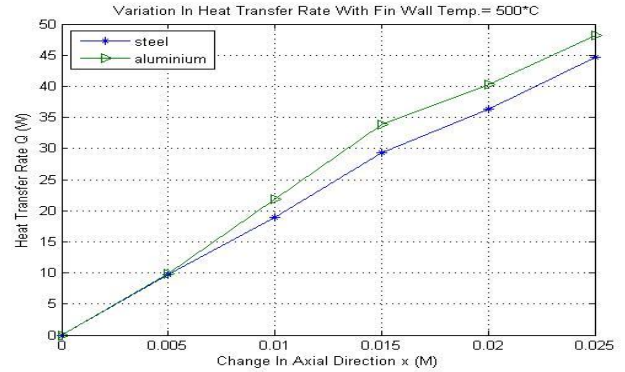
**Figure 5:** Variation in Heat Transfer Rate Q (W) at different Position of fin For Triangular fin



**Figure 6:** Variation in Heat Transfer Rate Q (W) at different Position of fin with  $h=90\text{ w/m}^2\text{-k}$  for Triangular fin



**Figure 7:** Variation in Heat Transfer Rate Q (W) at different Position of fin with width  $w=200\text{mm}$  for Triangular fin



**Figure 8:** Variation in Heat Transfer Rate Q (W) at different Position of fin with Fins wall temp  $T_0=500^0\text{C}$  triangular fin

**CONCLUSIONS**

A numerical study of straight fins are discussed. A Straight Triangular fin of steel and aluminum are used as working material. Key conclusion of this chapter can be summarized as follows:

Grid temperature and heat flux represent successfully the thermal behavior of the system. Fin wall temperature is decreasing along the length direction of Triangular fins and steel which has lower value of thermal conductivity (k) drop down more temperature. As we are using the same material of fins, Larger the surface area, larger the heat transfer rate Q (W) and heat flux (W/Kg). Higher the heat transfer rate Q (W), higher value of fin effectiveness and fin efficiency. Aluminum obtains larger value of fin efficiency and fin effectiveness then steel. Triangular fin contains less amount of material and high amount of heat flow rate per unit mass in comparison of rectangular fin. For this reason we should be use Triangular fin. Aluminum obtains larger value of heat flow rate per unit mass then steel. Thus triangular fin is more suitable for application that minimum weight such as space application.

**References**

- [1] Incropera, F. P., and DeWitt, D. P., "Fundamentals of Heat and Mass Transfer", 5th Ed.
- [2] Q.D. Kern, D.A. Kraus, Extended Surface Heat Transfer, McGraw-Hill, New York, 1972.
- [3] Kraus, A. D., Aziz, A, and Welty, J., Extended surface heat Transfer, John Wiley, 2001
- [2] E. Schmidt, Die Wärmeübertragung durch Rippen, Z. Ver. Dtsch. Ing. 70 (1926) 885–951.
- [3] R. Duffin, A variational problem relating to cooling fins, J. Math. Mech. 8 (1959) 47–56.
- [4] C.Y. Liu, A variational problem with applications to cooling fins, J. Soc. Ind. Appl. Math. 10 (1962) 19–27.
- [5] W.S. Minkler, The effects of internal heat generation on heat transfer in thin fins, Nucl. Sci. Eng. 7 (1960) 400406.

- [6] C.J. Maday, The minimum weight one dimensional straight fin, ASME J. Eng. Ind. 96 (1974) 161–165.
- [7] S. Guceri, C.J. Maday, A least weight circular cooling fins, ASME J. Eng. Ind. 97 (1975) 1190–1193.
- [8] A. Aziz, Optimum dimensions of extended surfaces operating in a convective environment, Appl. Mech. Rev. 45 (1992) 155–173.
- [9] W.M. Murray, Heat transfer through an annular disc or fin of uniform thickness, Trans. ASME J. Appl. Mech. 60(1938) A78.
- [10] K.A. Gardner, Efficiency of extended surface, Trans. ASME 67 (1945) 621–631.
- [11] B.T.F. Chung, R.W. Kan, The optimum dimensions of convective straight fins with profile curvature effect, AIChE J. 83 (1987)77–83.
- [12] P. Razelos, B.R. Satyaprakash, Analysis and optimization of convective trapezoidal profile longitudinal fins, ASME J. Heat Transfer 115 (1993) 461–463.
- [13] C. H Li, Optimum cylindrical pin fin, Int. J. Heat Mass Transfer 29 (1983) 1043–1044.
- [14] A. Sonn, A. Bar-cohen, Optimum cylindrical pin fin, ASME J. Heat Transfer 103 (1981) 814–815.
- [15] P. Razelos, The optimum dimensions of convective pin fins, ASME J. Heat Transfer 105 (1983) 411–413.
- [16] A. Ullmann, H. Kalman, Efficiency and optimized dimensions of annular fins of different cross-section shapes, Int. J. Heat Mass Transfer 32 (1989) 1105–1110.

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